Mathematics: analysis and approaches
Standard level
Paper 2

Name

Date: _____

1 hour 30 minutes

Instructions to candidates

- Write your name in the box above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written in the answer boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your name on each answer sheet and attach them to this examination paper.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

exam: 9 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A (37 marks)

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

An amount of £1000 is invested on January 6^{th} 2025 in a bank account with an interest rate of 2.5%, compounded annually. There are no further deposits or withdrawals and the annual interest earned is added to the account each year on January 6^{th}.

(a) Find the amount in the account after six years. [3]

(b) Find the number of years after which the account first exceeds £1500. [4]

2. [Maximum mark: 6]

Given that θ is an obtuse angle and $\sin\theta = \frac{3}{5}$, find the **exact** value of each of the following expressions:

(a) $\cos\theta$ [2]

(b) $\sin 2\theta$ [2]

(c) $\cos 2\theta$ [2]

3. [Maximum mark: 6]

Events A and B are such that $\mathrm{P}(A \cup B) = 0.9$, $\mathrm{P}(A \cap B) = 0.45$ and $\mathrm{P}(A \mid B) = 0.75$.

(a) Find $\mathrm{P}(B)$. [2]

(b) Find $\mathrm{P}(A)$. [2]

(c) Hence, show that events A and B are independent. [2]

4. [Maximum mark: 5]

Consider the function $y = p + \frac{p^2}{x} + x^2$, $x \neq 0$, where p is a constant.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(b) The graph of the function has a local minimum point at $(2,8)$. Find the value of p. [4]

5. [Maximum mark: 6]

A multiple choice test consists of twelve questions. Each question has four answers. Only one of the answers is correct. For each question, Emma randomly chooses one of the fours answers.

(a) Find the expected number of questions Emma answers correctly. [1]

(b) Find the probability that Emma answers exactly four questions correctly. [2]

(c) Find the probability that Emma answers more than four questions correctly. [3]

6. [Maximum mark: 7]

It is known that two out of five cups of coffee served at Bella's Coffee Shop contain more than 100 mg of caffeine. It is also known that four out of five cups served at Bella's contain more 85 mg of caffeine.

Assuming the amount of caffeine in a cup of coffee at Bella's is modelled by a normal distribution, find the mean and standard deviation of the caffeine content in a cup of coffee served at Bella's.

Do **not** write solutions on this page.

Section B (43 marks)

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

7. [Maximum mark: 12]

A company that manufactures car tires conducts an experiment to determine how a certain type of tire maintains its air pressure over time. A new tire is fitted to a wheel. The tire is then inflated to its recommended pressure of 39 psi (pounds per square inch) and the tire is placed in a temperature controlled room. At three-month intervals, the air pressure of the tire is measured giving the following results.

time (x months)	0	3	6	9	12	15	18	21	24
tire pressure (y psi)	39.0	37.2	35.6	34.7	33.5	32.2	30.6	29.2	28.1

(a) Write down the equation of a straight line model for the association between time and tire pressure, i.e. an equation of the regression line of y (pressure) on x (time). [2]

(b) Comment on the strength of the association between time and tire pressure. [2]

(c) Use your straight line (linear regression) model to interpret the meaning of

(i) the gradient

(ii) the y-intercept. [2]

(d) Estimate the air pressure (psi) of the tire 20 months after being fitted to the wheel. [2]

(e) Do not give numerical answers for this question. Comment on the appropriateness of using your model to:

(i) estimate the tire pressure after three years;

(ii) estimate the number of months it would take for the tire pressure to decrease to 30 psi. [4]

8. [Maximum mark: 14]

The diagram below shows a circle with centre O and radius 6 cm.

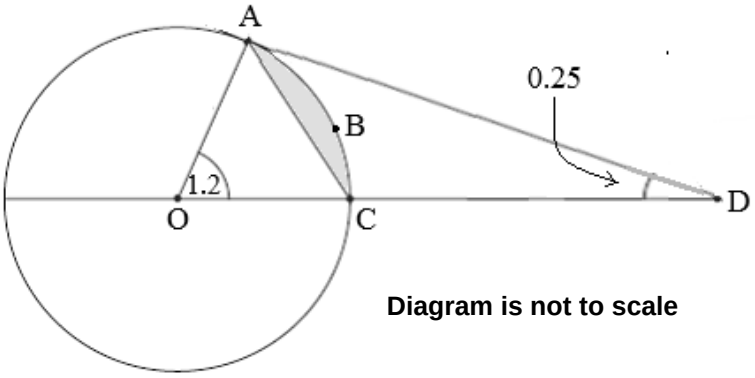


Diagram is not to scale

The points A, B and C lie on the circle. The point D is outside the circle and lies on (OC). Angle $\mathrm{AOC} = 1.2$ radians and angle $\mathrm{ADO} = 0.25$ radians.

(a) Find the area of the sector OABC. [3]

(b) Find the area of the shaded region bounded by the chord AC and the arc ABC. [4]

(c) Find AD. [3]

(d) Find OD [4]

9. [Maximum mark: 17]

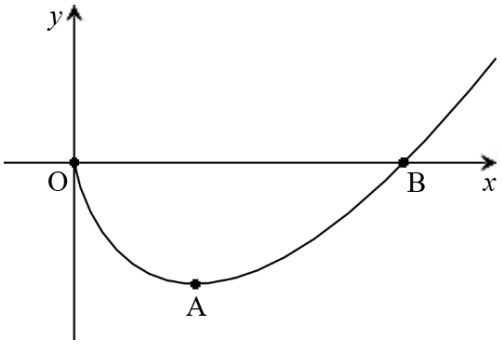
Consider the function $g_n(x) = \begin{cases} x\ln x - nx, & x > 0 \\ 0, & x = 0 \end{cases}$, where $n = 0, 1, 2, \ldots$

(a) Find the derivative of $g_n(x)$, $x > 0$.

[2]

The graph of the function $g_n(x)$ is shown.

One x-intercept is at the origin O and the other x-intercept is at point B. The graph has an absolute minimum at point A.



(b) Show that the x-coordinate of A is e^{n-1}. [2]

(c) Find the x-coordinate of B. [3]

(d) Find the equation of the tangent to the graph of $g_n(x)$ at B. [3]

(e) Find the area bounded by the graph of $g_n(x)$ and the x-axis when $n = 1$. [3]

(f) Show that the x-coordinates of the minimum point on the graph of $g_n(x)$, for consecutive values of n, form a geometric sequence. [4]